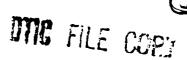


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IN A DENSITY GRADIENT

G. J. MORALES AND J. E. MAGGS

PPG-1130

February 1988

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## Heating Efficiency of Beat-Wave Excitation in a Density Gradient

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A simple model is presented which yields analytical expressions for the heating efficiency of beat-wave excitation in a plasma with a linear density profile. The effect of self-consistent Landau damping by tail electrons is included without recourse to WKB approximations.



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Presently there is renewed interest in beat-wave excitation l-4 of Langmuir waves to generate fast electrons, two possible applications being advanced particle accelerators and modification of the earth's ionosphere. Although the description of this process in uniform plasmas is relatively well understood, the presence of a density gradient introduces several complications which have been handled in a WKB sense in the pioneer work by Rosenbluth and Liu<sup>2</sup> and in computer simulations by Cohen, et al<sup>4</sup>. Because in certain experiments the role of density gradients may not be negligible or in fact may be useful in achieving a desired result, it is convenient to obtain analytical expressions which explicitly exhibit the dependence of the heating efficiency on the density scale length L. In this brief note we present a relatively simple model which yields such a result and incorporates the self-consistent effect of Landau damping by fast electrons without recourse to WKB approximations. It is hoped that this result may be useful in estimating the performance of future experiments and that the model may provide a basis for more elaborate calculations of analogous phenomena in nonuniform plasmas.

The one-dimensional model consists of separating the roles of warm background electrons having a spatially varying density  $n_0(z)$ , and fast tail electrons whose density  $n_t$  is considered to be uniform (because of their longer collision mean free path) over the region where the relevant wave-particle interactions occur. The tail density is assumed small ( $n_t/n_0 << 1$ ) so that the principal contribution of the fast particles enters through Landau damping, while the warm background electrons determine the propagation features of the driven Langmuir wave. The background electrons are created as a warm fluid with thermal velocity  $\overline{v}$  and the tail electrons are described kinetically assuming an unperturbed tail distribution

$$f_{ot} = \frac{n_t}{(2\pi v_t^2)^{1/2}} \exp[-v^2/2v_t^2]$$
 (1)

Beat excitation is envisioned to arise from the beat ponderomotive force acting on the background electrons and caused by two transparent electromagnetic waves having frequencies and wave numbers  $\omega_j$ ,  $k_j$ ; j=1,2. These waves propagate along the nonuniform direction z and can individually move in the direction of decreasing density  $(k_j > 0)$  or towards increasing density  $(k_j < 0)$ . The self-consistent beat excited longitudinal electric field has a harmonic time dependence E(z) exp(-i $\omega$ t) with  $\omega = \omega_2 - \omega_1$ , and is determined from Poisson's equation

$$\frac{\partial}{\partial z} \left[ E(z) - E_0(z) \right] = -4\pi e \left[ \tilde{n}(z) + \tilde{n}_t(z) \right] , \qquad (2)$$

in which the effective pump source  $E_{0}(z)$  for beat excitation is

$$E_0 = -\left(\frac{e}{mc}\right) \left(\frac{\omega}{\omega_1 \omega_2}\right) E_{01}^* E_{02} \qquad , \tag{3}$$

where  $E_{0j}$  is the complex amplitude of the jth electromagnetic wave, e and m are the charge and mass of an electron, and c is the speed of light. In Eq. (3) the frequencies of the electromagnetic waves are assumed high enough that their phase velocities are essentially the speed of light. The perturbed charge densities (oscillating at frequency  $\omega$ ) for the background  $n_0$  and tail  $n_0$  are calculated from the fluid and Viasov equations, respectively.

Introducing the Fourier transform of the driven field

$$E(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \ \widetilde{E}(k) e^{ikz} , \qquad (4)$$

and assuming a linear density profile with scale length L, i.e.,  $1-\omega_p^2(z)/\omega^2=z/L, \text{ where } \omega_p(z) \text{ is the local electron plasma frequency,}$  transforms Eq. (2) into a differential equation in k-space

$$\frac{i}{L}\frac{d}{dk}\widetilde{E}(k) + \left[i\frac{d}{dk}g(k) - \frac{3k^2}{kD^2}\right]\widetilde{E}(k) = 2\pi E_0 \delta(k - k_b) \qquad , \qquad (5)$$

with  $k_b$  =  $k_2$  -  $k_1$  the beat wave number,  $k_D$  =  $\omega/\overline{v}$  , and

$$g(k) = \sqrt{\frac{\pi}{2}} \left( \frac{n_t}{n_0} \right) \left( \frac{\bar{v}}{v_t} \right) k_D \, sgn(k) exp[-\frac{\omega^2}{2k^2 v_t^2}]$$
, (6)

where sgn(k) = 1 for k > 0, -1 for k < 0.

Equation (5) can be integrated to yield the spectrum

$$\tilde{E}(k) = -2\pi i E_0 L\theta(k - k_b) exp[F(k) - F(k_b)] \qquad , \qquad (7)$$

$$F(k) = -i \frac{k^3 L}{k_D^2} - g(k)L \qquad , \qquad (8)$$

where  $\theta(k)$  is the Heaviside step - function.

The sign of  $k_b$  determines the direction in which the directly driven Langmuir wave propagates. For  $k_b>0$ , propagation occurs in the direction of decreasing density and results in a single wave whose phase velocity decreases as z increases (in the WKB sense as  $\sqrt{3}$   $\overline{v}$  (z/L) ). For  $k_b<0$  the spatial pattern<sup>6</sup> consists of two waves. The directly driven wave propagates towards increasing density and upon reaching the cut-off point (z=0 where  $\omega_p=\omega$ ) exhibits partial (because of Landau damping) reflection and thus generates a second wave which behaves analogously to the one directly driven for  $k_b>0$ . These features imply that for  $k_b>0$ , tail electrons with velocity v>0 are

accelerated and this results in an increase of the heat flux  $\delta Q_+$  as  $z + \infty$ . However, for  $k_b < 0$ , tail modification occurs for both v < 0 and v > 0 particles, and causes heat flux modifications in both directions. The self-consistent modifications in the time-averaged tail distribution function, accurate to second-order in the amplitude of the longitudinal electric field, are

$$\langle \delta f_{t}^{\pm} (z = \pm \infty, v) \rangle = (\frac{e}{2m})^{2} \frac{1}{v} \frac{\partial}{\partial v} \{ |\widetilde{E} (k = \frac{\omega}{v})|^{2} \frac{1}{v} \frac{\partial}{\partial v} f_{ot}^{\pm} \}, \qquad (9)$$

where the label  $\pm$  refers to particles moving in the positive or negative z direction.

In determining the efficiency of energy transfer to the plasma the quantities of interest are the asymptotic heat fluxes

$$\delta Q_{\pm} = \pm \int_{0}^{\pm \infty} dv \frac{mv^{3}}{2} < \delta f_{\pm}^{\pm} > \qquad , \qquad (10)$$

which using Eq. (9) and integrating by parts can be put in the form

$$\delta Q_{\pm} = \pm \frac{e^2}{4m} \int_0^{\pm \infty} dv \left| \widetilde{E}(k = \frac{\omega}{v}) \right|^2 \frac{\partial}{\partial v} f_{ot}^{\pm}(v) . \qquad (11)$$

From Eq.(6) it is seen that  $g(k = \omega/v)$  is proportional to  $f_{ot}(v)$  and hence  $|\tilde{E}(k = \omega/v)|^2$  depends exponentially on  $f_{ot}(v)$ . This implies that the integral in Eq.(11) can be evaluated analytically without approximations.

Defining the heating efficiency  $\eta_{\pm}$  as the ratio of the enhanced tail heat flux to the Poynting flux  $P_j = c |E_j|^2/8\pi$  of one (say j=1) of the electromagnetic waves, it is found that for direct excitation with  $k_b>0$ 

$$\eta_{+} \equiv \frac{\delta Q_{+}}{P_{1}} = 2\pi \frac{\omega^{5}}{\omega_{1}^{2} \omega_{2}^{2}} \left(\frac{L}{c}\right) \left(\frac{P_{2}}{n_{0}mc^{3}}\right) \left[1 - \exp\left[-2\alpha(1 - e^{-\xi^{2}})\right]\right], \quad (12)$$

where  $\alpha=(\pi/2)^{1/2}$   $(n_t/n_0)$   $(\bar{v}/v_t)$   $(k_DL)$  measures the combined effect of tail density and scale length, and  $\xi\equiv\omega/(\sqrt{2}-k_Dv_t)$ . In Eq. (12) the background density  $n_0$  is evaluated at z=0.

In the limit of large  $\alpha(i.e., long scale length)$ 

$$\eta_{+} \approx 2\pi \left(\frac{\omega}{\omega_{j}}\right)^{5} \left(k_{0j}L\right) \left(\frac{P_{2}}{n_{0}mc^{2}}\right) \equiv \eta_{0}$$
 (13)

where  $k_{0j} = \omega_j/c$  since  $\omega_1 \approx \omega_2 >> \omega$ . It should be noted that in this limit the heating efficiency is independent of tail parameters. In the opposite limit, i.e.,  $\alpha << 1$ , corresponding to small tail densities and/or short scale lengths,

$$n_{+} \approx n_{0}\alpha(1 - e^{-\xi^{2}})$$
 (14)

showing a scaling proportional to  $n_t$  and  $L^2$ .

For excitation towards increasing density, i.e.,  $k_b < 0$ , the contributions from the two heat fluxes at  $z = \pm \infty$  must be included in evaluating the efficiency, which yields

$$\eta_{-} \equiv \frac{\delta Q_{+} - \delta Q_{-}}{P_{1}}$$

$$= \eta_{0} \left\{ 1 - \exp[-2\alpha(1 + e^{-\xi^{2}})] \right\} , \quad (15)$$

and for large  $\alpha$  reduces to  $\eta_- \approx \eta_0$ , as in the  $k_b > 0$  case. In the small  $\alpha$  limit Eq.(15) becomes

$$\eta_{-} \approx \eta_{0} \alpha (1 + e^{-\xi^{2}})$$
 (16)

For transparent electromagnetic waves propagating in the same direction (i.e.,  $\underline{k}_1 \cdot \underline{k}_2 > 0$ ) it is likely that in many applications  $\xi >> 1$ . In this special case it is found that  $\delta(\xi_+ + 0)$ , and thus

$$\eta_{+} = \eta_{-} \approx \eta_{0} (1 - \exp(-2\alpha))$$
 (17)

The physics behind Eq.(17) is that particle acceleration results from the transit-time of v > 0 particles through the spatially varying envelope (and not the phase) of the beat excited wave.

In summary, a simple model has been developed that yields a compact analytic expression

$$n_{\pm} = n_0 \left\{ 1 - \exp\left[-2\alpha(1 \mp e^{-\xi^2})\right] \right\}$$
, (18)

for the heating efficiency of beat-wave excitation in the direction of decreasing (+) and increasing (-) density in a nonuniform plasma. This result includes nonuniform wave propagation and self-consistent Landau damping without recourse to WKB approximations.

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## References

- 1. B. Cohen, A.N. Kaufman, and K. Watson, Phys. Rev. Lett. 29, 581 (1972).
- 2. M.N. Rosenbluth and C.S. Liu, Phys. Rev. Lett. 29, 701 (1972).
- 3. A.N. Kaufman and B. Cohen, Phys. Rev. Lett 30, 1306 (1973).
- 4. B. I. Cohen, M.A. Mostrom, D.R. Nicholson, A.N. Kaufman, and C. Max, Phys. Fluids 18, 470 (1975).
- 5. For an overview of this rapidly advancing field consult Laser Acceleration of Particles, AIP Conf. Proc. No 130, C. Joshi and T. Katsouleas, eds. (AIP, New York, 1985).
- 6. Detailed analysis and graphs of the spatial pattern of beat excitation in a density gradient are given by G.J. Morales, M.M. Shoucri, and J.E. Maggs, UCLA Center for Plasma Physics and Fusion Engineering report PPG-1089, August, 1987 (submitted to Phys. Fluids).

- PPG-1106 "Nonlinear Resonance of Two-Dimensional Ion Layers", S.A. Prasad and G.J. Morales, submitted to The Physics of Fluids, September, 1987.
- PPG-1107 "Laser Accelerators," Francis F. Chen, to be published in the Handbook of Plasma Physics, October, 1987.
- PPG-1108 "Ion Bernstein Modes in Current-Carrying Plasmas," J. L. Milovich, Ph.D. dissertation, October, 1987.
- PPG-1109 "Optimum Rankine Cycle for High Power Density Fusion Reactor Using Liquid Metal as Primary Coolant", M. Hasan, D. Sze, Report, January, 1988.
- PPG-1110 "Titan Reverse-Field Pinch Fusion Reactor Study," F. Najmabadi, et al., present at IEEE 12th Symposium on Fusion Engineering, October 12-16, 1987, Monterey, Calif., October, 1987.
- PPG-1111 "Revised Benchmark Timings with Particle Plasma Simulation Codes," V.K. Decyk, (revision of PPG-950), October 1987.
- PPG-1112 "Gain and Bandwidth of the Gyro-TWT and Carm Amplifier," K.R. Chu and A.T. Lin, October 1987.
- PPG-1113 "Detailed Experimental Observations of the Tearing of an Electron Current Sheet", Walter Gekelman and Hans Pfister, submitted to Physics of Fluids, October 1987.
- PPG-1114 "Instability of the Sheath Plasma Resonance", R.L. Stenzel, submitted to Physics of Fluids, October 1987.
- PPG-1115 "Experimental Study of Time-Varying Current Flow Between Electrodes Immersed in a Laboratory Magnetoplasma," J. Manuel Urrutia, Ph.D. Dissertation, October 1987.
- PPG-1116 "Counterstreaming Electron-Beam Beat-Wave Accelerator," Y.T. Yan, C.J. McKinstrie, T. Katsouleas, and J.M. Dawson, accepted for publication in Phys. Review A (December 1987), October 1987.
- PPG-1117 "The TITAN Reversed-field Pinch Fusion Reactor Study", R.W. Conn, F. Najmabadi, and the TITAN research group, submitted for Proc. of IEEE 12th Symposium on Fusion Engineering, Monterey, CA, Oct. 12-16, 1987, November 1987.
- PPG-1118 "Comparative Study of Cross-Field and Field Aligned Electron Beams in Experiments," R.M.Winglee and P.L.Pritchett, submitted to Journal of Geophysical Research, November 1987.
- PPG-1119 "Theory, Design, and Operation of High Harmonic Gyro-Amplifiers" David F. Furuno, Ph.d disseration, December 1987.
- PPG-1120 "Fluid Theories of Slow Shock Structure", C.F. Kennel, December 1987.

- PPG-1121 "Plasma Acceleration of Particle Beams," T. Katsouleas and J.M. Dawson, to appear in Physics of Particle Accelerators, Vol. V., 1988 (Jan. 1988).
- PPG-1122 "Diamagnetic Drift Effects on Ion Rernstein Mode Propagation in a Plasma Slab," R. D. Ferraro and B. D. Fried, submitted to Phys. Fluids, Jannuary 1988.
- PPG-1123 "Hydrogen Pumping and Release by Graphite Under High Flux Plasma Bombardment," Y. Hirooka, W.K. Leung, R.W. Conn, D.M. Goebel, B. Labombard, R. Nygren, K.L. Wilson submitted to Journal of Vaccum Science and Technology, January 1988.
- PPG-1124 "1987 Research Highlights in The Pisces Program," R.W. Conn, et al, January 1988.
- PPG-1125 "Magnetic Fusion Energy, vol. 5. Technical Assessment of Critical Issues in the Steady State Operation of Fusion Confinement Devices," D. M. Goebel, Assessment Chairman, et al, January 1988.
- PPG-1126 "A Particle MHD Simulation Approach with Application to a Global Comet-Solar Wind Interaction Model," R. Sydora and J. Raeder, January 1988, sub. to Cometary and Solar Plasma Physics.
- PPG-1127 "Particle Collector Scoops for Improved Exhaust In 'Axi-semetric' Devices," R.W. Conn and G.H. Wolf, November 1987, sub. to Fusion Eng. and Design (2/88).
- PPG-1128 "Fusion Core Start-Up, Ignition and Burn Simulations of Reverse-Field Pinch (R.F.P.) Reactors," Yuh-Yi Chu, (Ph.D. Dissertation), 2/88.
- PPG-1129 "Nonlinear, Dispersive, Elliptically Polarized Alfven Waves," C.F. Kennel, B. Buti, and R. Pellat, submitted to Phys. of Fluids, February 1988.
- PPG-1130 "Heating Efficiency of Beat-Wave Excitation in a Density Gradient," G.J. Morales and J.E. Maggs, February 1988.
- PPG-1131 "Electrostatic Whistler Mode Conversion at Plasma Resonance," J.E. Maggs and G.J. Morales, February 1988.
- PPG-1132 "Shock Structure in Classical Magnetohydrodynamics", C.F. Kennel, sub. to JGR, February 1988.
- PPG-1133 "Generation and Propagation of Kilometric Radiation in the Auroral Plasma Cavity, P.L. Pritchett and R.M. Winglee, sub. to JGR, February 1988.

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